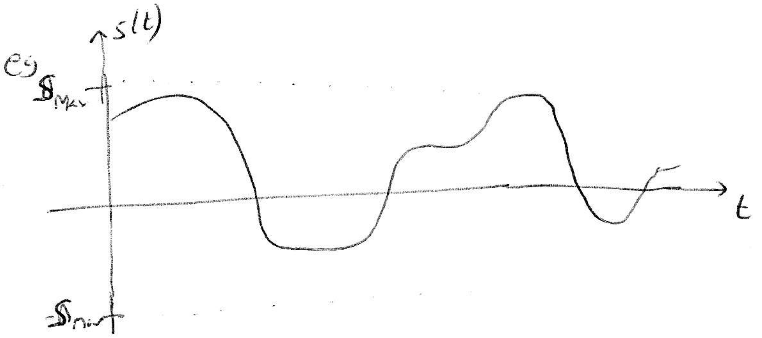
Modulation

L2

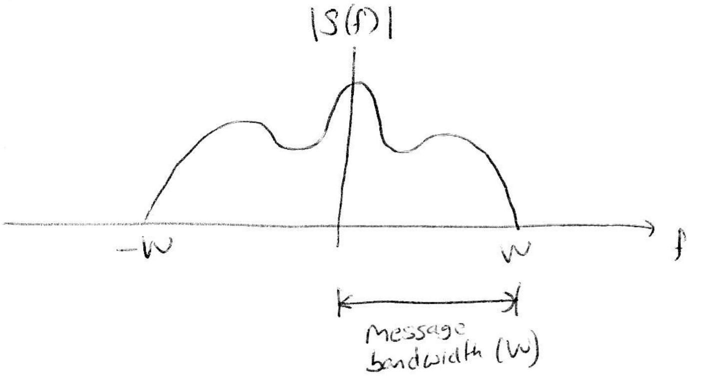
Want to transmit an analogue baseband signal, $s(t)$.

eg



- amplitude limited to $|s(t)| \leq S_{max}$
- zero mean (no DC)

Spectrum $|S(f)|$ with $S(f) = \mathcal{F}\{s(t)\}$



bandlimited to W Hz

$|S(f)| \cong 0 \quad \forall \ |f| > W$

Average Power $P_s = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |s(t)|^2 \, dt$

For electrical transmission over short distances, use $s(t)$ to directly control line voltage.

Many communication channels do not support baseband transmission

eg.
- phone network blocks DC (0 Hz) signal components
- fibre optic channels work with light
- long antennas needed for low frequencies
- wireless spectrum is shared (each user is given a separate frequency band)

Use Modulation to generate a bandpass signal from a baseband one.

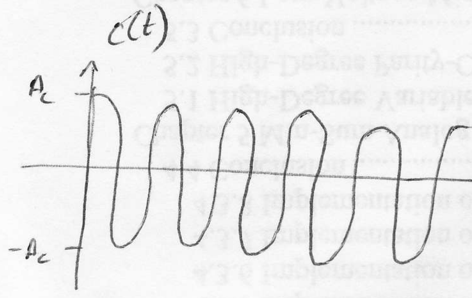
Amplitude Modulation (AM)

Use $s(t)$ to modulate (vary) the amplitude of a sinusoidal carrier wave.

$c(t) = A_c \cos(2\pi f_c t)$

A_c = carrier amplitude

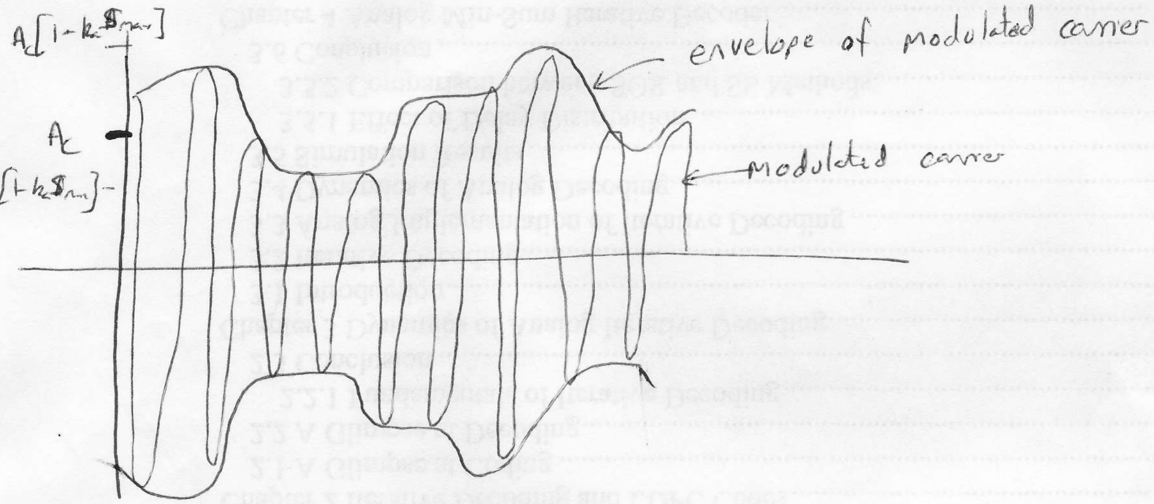
f_c = carrier frequency ($f_c \gg W$)



Double-sideband large-carrier AM (DSB-LC)

$s_c(t) = A_c[1 + k_a s(t)]\cos(2\pi f_c t)$

k_a = amplitude sensitivity



k_a is typically chosen so that $1 - k_a s_{max} > 0 \Rightarrow k_a s_{max} < 1$

$\mu = k_a s_{max}$ ← modulation factor ← measures how much modulated signal varies about its original value

If $\mu > 1$ then the signal is overmodulated

Spectrum

$$S_c(f) = \mathcal{F}\{s_c(t)\} = \int_{-\infty}^{\infty} s_c(t) e^{-j2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} A_c[1 + k_a s(t)] \cos 2\pi f_c t \, e^{-j2\pi ft} dt$$

$$\cos A = \frac{1}{2}[e^{jA} + e^{-jA}]$$

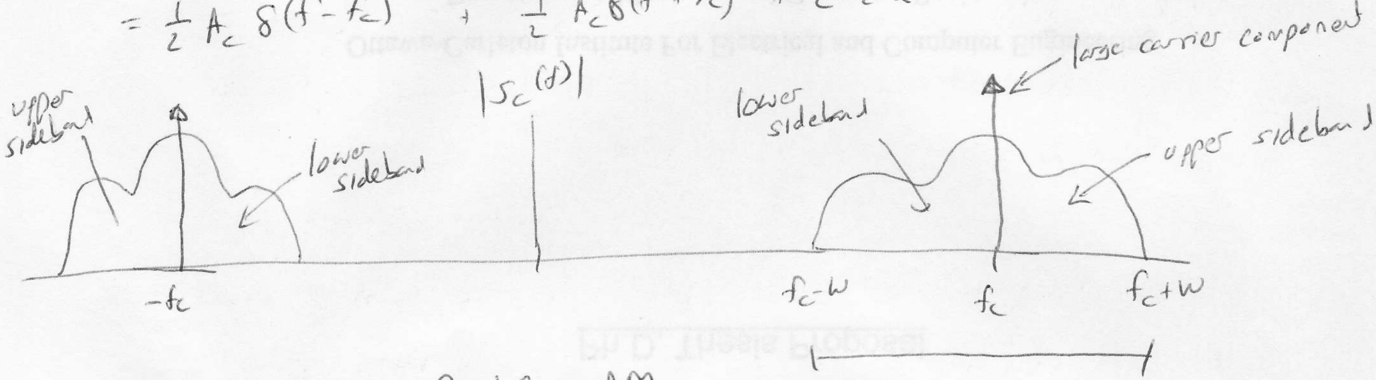
$$\int_{-\infty}^{\infty} e^{-j2\pi(f-f_0)t} dt = \delta(f-f_0)$$

$$= A_c \int_{-\infty}^{\infty} [1 + k_a s(t)] \frac{1}{2}[e^{j2\pi f_c t} + e^{-j2\pi f_c t}] dt \, e^{-j2\pi ft} dt$$

$$= \frac{1}{2}A_c \int_{-\infty}^{\infty} e^{-j2\pi(f-f_c)t} dt + \frac{1}{2}A_c \int e^{-j2\pi(f+f_c)t} dt + \frac{1}{2}A_c k_a \int s(t) e^{-j2\pi(f-f_c)t} dt$$

$$+ \frac{1}{2}A_c k_a \int s(t) e^{-j2\pi(f+f_c)t} dt$$

$$= \frac{1}{2}A_c \delta(f-f_c) + \frac{1}{2}A_c \delta(f+f_c) + \frac{1}{2}A_c k_a S(f-f_c) + \frac{1}{2}A_c k_a S(f+f_c)$$



DSB-LC AM

transmission bandwidth $B_T = 2W$

(double sideband)

Average transmitted Power

$$P_c = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |s_c(t)|^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A_c^2\left[1 + k_a s(t)\right]^2 \cos^2 2\pi f_c t\,dt \qquad \cos^2 A = \frac{1}{2}\left[1+\cos 2A\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A_c\left[1 + 2k_a s(t) + k_a^2 s^2(t)\right]\frac{1}{2}\left[1+\cos 4\pi f_c t\right]dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \frac{A_c^2}{2}\Big[1 + 2k_a s(t) + k_a^2 s^2(t) + \underbrace{\cos 4\pi f_c t + 2k_a s(t)\cos 4\pi f_c t + k_a^2 s^2(t)\cos 4\pi f_c t}_{\text{tends towards zero}}\Big]dt$$

$$= \frac{A_c^2}{2}\left[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}dt\right] + \underbrace{A_c^2 k_a\left[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}s(t)\,dt\right]}_{=0 \text{ if } s(t) \text{ has zero mean}} + \frac{A_c^2}{2}k_a^2\left[\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}s^2(t)\,dt\right]$$

$$= \underbrace{\frac{1}{2}A_c^2}_{\text{power of the carrier}} + \underbrace{\frac{1}{2}A_c^2 k_a^2 P_s}_{\text{power of the message signal}}$$

Modulation Efficiency
- % of total power that conveys information

$$E = \frac{\frac{1}{2}A_c^2 k_a^2 P_s}{\frac{1}{2}A_c^2 k_a^2 P_s + \frac{1}{2}A_c^2}\times 100\% = \frac{k_a^2 P_s}{1 + k_a^2 P_s}\times 100\%$$

As $k_a \downarrow$, $E \downarrow$

As $k_a \to \infty$, $E \to 100\%$

Note: DSB-LC AM is
1) Wasteful of power ($E < 100\%$)
2) Wasteful of bandwidth (don't need both sidebands - one is redundant)
3) ~~Cheap~~ Inexpensive to implement

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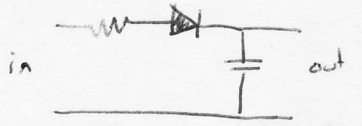
transmitted codeword or the transmitted bit correctly, or equivalently, what the average probability of error for SOR is. Focusing on error probability as the main measure of performance, we show that for the cases that we considered here (BP in LLR domain and MS in LLR domain) the optimal value for β is always less than one. We also observe that for optimal β, SOR outperforms SS method. The improvement in performance (which is the result of an increase in the number of converged cases) increases with the number of iterations, and in the limit as the number of iterations goes to infinity, the best performance for SOR is obtained by tending the step size (and the relaxation factor) to zero (This corresponds to the steady-state performance of analog decoding).

Adopting alternate numerical methods to improve convergence in iterative decoding of turbo codes was proposed in [44] and was also studied in [40]. In [40], the authors identified turbo decoding as fixed point iteration and examined alternative numerical methods including SOR for solving the corresponding fixed-point problem. The final conclusion of [40], however was that for turbo codes the conventional successive substitution is the superior choice in most situations. This is in clear contrast with our results for LDPC codes, where SOR in particular provides considerable improvement over SS for both BP and MS algorithms. On the same theme of research, in the past few years and mainly in the context of short low-density parity-check (LDPC) codes, there have been many attempts for improving the performance of iterative decoding algorithms. We have noted that many of these proposed methods can be interpreted as the application of different iterative numerical methods for solving the fixed-point problem in hand. For instance, in [33]-[36], variants of "damped substitution method" [40] have been used for improving the performance of MS and sum-product (SP) iterative decoding algorithms.

Envelope Detector

- rectify and apply Lowpass filter (LPF)



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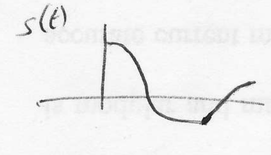
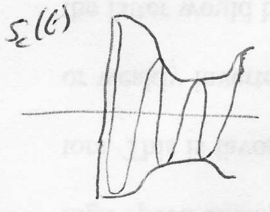
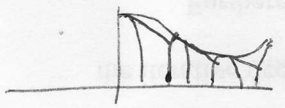
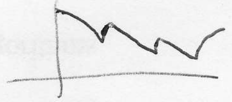
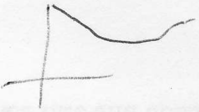


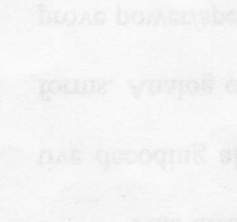








- does no
- easy to implement
- does not work if signal is overmodulated



envelope