Identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$$\sin A = \frac{1}{j2}\left(e^{jA} - e^{-jA}\right)$$

$$\cos A = \frac{1}{2}\left(e^{jA} + e^{-jA}\right)$$

$e^{\pm jA} = \cos A \pm j \sin A$

$$\sum_{m=-\infty}^{\infty} e^{-j2\pi f m T} = \frac{1}{T}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$

$$\int_{-\infty}^{\infty} e^{j2\pi f t}\, dt = \delta(f)$$

Some Fourier Transform Pairs

$h(t)$		$H(f)$
$\text{rect}\left(\frac{t}{T}\right)$	$\rightarrow$	$T\frac{\sin \pi f T}{\pi f T}$
$\cos(2\pi f_0 t)$	$\rightarrow$	$\frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)$
$u(t)$	$\rightarrow$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$e^{-at}u(t)$	$\rightarrow$	$\frac{1}{a + j2\pi f}$
$te^{-at}u(t)$	$\rightarrow$	$\frac{1}{(a + j2\pi f)^2}$
$e^{-a\lvert t\rvert}$	$\rightarrow$	$\frac{2a}{a^2 + (2\pi f)^2}$
$e^{-t^2/(2\sigma^2)}$	$\rightarrow$	$\sqrt{2\pi\sigma^2}e^{-2\pi^2 f^2 \sigma^2}$
$\delta(t - t_0)$	$\rightarrow$	$e^{-j2\pi f t_0}$
$\frac{\sin 2\pi W t}{2\pi W t}$	$\rightarrow$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\sum_{m=-\infty}^{\infty} \delta(t - mT)$	$\rightarrow$	$\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T}\right)$

Properties of the Fourier Transform

Operation	$h(t)$	$H(f)$
Linearity	$a_1 h_1(t) + a_2 h_2(t)$	$a_1 H_1(f) + a_2 H_2(f)$
Duality	$H(t)$	$h(-f)$
Complex Conjugate	$h^*(t)$	$H^*(-f)$
Time Scaling	$h(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}H(\frac{f}{\lvert\alpha\rvert})$
Time Shift	$h(t - t_0)$	$H(f)e^{-j2\pi f t_0}$
Time Convolution	$\int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)\, d\tau$	$H_1(f)H_2(f)$
Time Multiplication	$h_1(t)h_2(t)$	$\int_{-\infty}^{\infty} H_1(u)H_2(f-u)\, du$
Time Differentiation	$\frac{d}{dt}h(t)$	$j2\pi f H(f)$
Time Integration	$\int_{-\infty}^{t} h(\tau)\, d\tau$	$\frac{1}{j2\pi f}H(f) + \frac{H(0)}{2}\delta(f)$
Frequency Translation	$h(t)e^{j2\pi f_0 t}$	$H(f - f_0)$
Amplitude Modulation	$h(t)\cos(2\pi f_0 t)$	$\frac{1}{2}H(f-f_0) + \frac{1}{2}H(f+f_0)$

Indefinite Integrals

$$\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b)$$

$$\int \cos(ax+b)\, dx = \frac{1}{a}\sin(ax+b)$$

$$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int \sin ax \cos ax\, dx = \frac{1}{2a}\sin^2 ax$$

$$\int \sin ax \sin bx\, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}$$

$$\int \cos ax \cos bx\, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}$$

$$\int \sin ax \cos bx\, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

$$\int \cos ax \sin bx\, dx = \frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$$

$$\int x \sin ax\, dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x \cos ax\, dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2 \sin ax\, dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2 \cos ax\, dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int e^{ax}\, dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\, dx = \frac{1}{a^2}e^{ax}(ax - 1)$$

$$\int x^2 e^{ax}\, dx = \frac{1}{a^3}e^{ax}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\, dx = \frac{1}{a^2+b^2}e^{ax}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\, dx = \frac{1}{a^2+b^2}e^{ax}(a\cos bx + b\sin bx)$$

$$\int \left[\frac{\sin ax}{x}\right]^2 dx = a\int \frac{\sin 2ax}{x}\, dx - \frac{\sin^2 ax}{x}$$

$$\int \ln x\, dx = x\ln x - x$$