Carleton University
Department of Systems and Computer Engineering

SYSC 3503 | **Communication Theory II** | **Winter 2016/17**

Assignment #1
Due on Thursday, January 19, 2017

1. The output signal, $y[n]$, of a system is

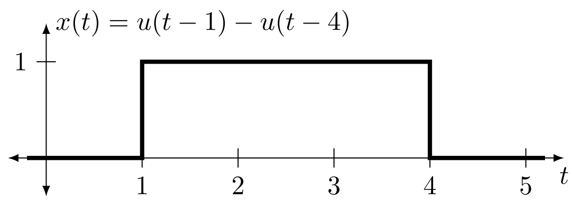
$$y[n] = (x[n])^2 - 2x[n-1] + x[n-2]$$

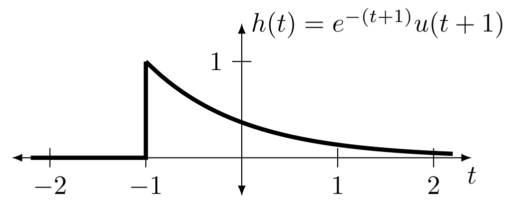
when the input signal is $x[n]$.

(a) Is the system memoryless or with memory?
(b) Is the system time-variant or time-invariant?
(c) Is the system linear or non-linear?
(d) Is the system causal?
(e) Is the system stable?
(f) Sketch the impulse response.

Justify all your answers.

2. Suppose signal $x(t)$, shown below, is passed through a linear time-invariant system with impulse response $h(t)$, also shown below, where $u(t)$ is the unit step function. Let $y(t)$ denote the output of the system.





(a) Find and sketch $y(t)$.
(b) Find a simple mathematical expression for $X(f) = \mathcal{F}\{x(t)\}$.
(c) Find a simple mathematical expression for $H(f) = \mathcal{F}\{h(t)\}$.
(d) Find a simple mathematical expression for $Y(f) = \mathcal{F}\{y(t)\}$.

3. Consider the discrete-time signal

$$x[n] = 4\delta[n] - 4\delta[n-1] + \delta[n-2] \ .$$

(a) Sketch $x[n]$.
(b) Find a simple mathematical expression for the $X(f)$, the discrete-time Fourier transform of $x[n]$.
(c) Sketch $|X(f)|$.
(d) Find all the values of the 4-point discrete Fourier transform of $x[n]$.

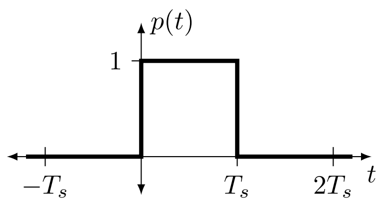
4. Suppose a signal, $x(t)$, is sampled at a rate of $f_s = 1/T_s$ samples per second. The resulting sample sequence, $\{x[n]\}$, is used to modulate the amplitude of a rectangular pulse train, giving signal

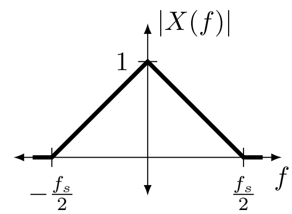
$$x_p(t) = \sum_{n=-\infty}^{\infty} x[n]p(t - nT_s)$$

where $p(t)$ is the rectangular pulse shown below.

(a) Find $X_p(f) = \mathcal{F}\{x_p(t)\}$. Express your answer in terms of $X(f) = \mathcal{F}\{x(t)\}$ and $P(f) = \mathcal{F}\{p(t)\}$.

(b) If $|X(f)|$ is as shown below, sketch $|X_p(f)|$.





5. Suppose the signal $x(t) = 2 + \cos(2\pi f_0 t)$ is used to modulate the amplitude of the carrier wave $y(t) = \cos(2\pi f_c t)$, giving signal $z(t) = x(t)y(t)$. Assume $f_c = 3f_0$.

(a) Sketch one period of $z(t)$.

(b) Sketch $Z(f) = \mathcal{F}\{z(t)\}$.

6. Prove Parseval's Theorem for continuous-time energy signals. That is, show that

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

where $X(f) = \mathcal{F}\{x(t)\}$.