

Problem Set #3

- **Textbook:** Ch. 4: 4, 10, 14, 22, 25, 32, 46, 48, 51, 61, 78, 81.

- **Supplementary:**

1 Suppose that X_1, \dots, X_n are independent and all have the cdf $F_X(x)$. Find the cdf of $Y = \max\{X_1, \dots, X_N\}$, and the cdf of $Z = \min\{X_1, \dots, X_N\}$.

2 Suppose that X_1 and X_2 have the joint density

$$f(x_1, x_2) = \begin{cases} 1 & \text{for } 0 < x_1, x_2 < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the joint density of $Y_1 = X_1/X_2$ and $Y_2 = X_1X_2$.

3 (a) A miner is trapped in a mine with 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him back to the mine after 5 hours of travel. The third door leads to a tunnel that will return him back to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety?

(b) Suppose that X and Y have joint density $f(x, y) = (1/2)e^{-y}$, for $y \geq 0$ and $-y \leq x \leq y$. Compute $P[X \leq 1 | Y = 3]$.