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Stochastic Processes 94553

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Problem Set #2

- **Textbook:** Ch. 3: 27, 37, 45, 59.
- **Supplementary:**

1 Let X denote the life time (in hundreds of hours) of a certain type of electronic component. These components frequently fail immediately upon insertion into the system. It has been observed that the probability of immediate failure is 1/4. If a component does not fail immediately, the life-length distribution has the exponential density:

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the distribution function for X and evaluate $P(X > 10)$.

2 A certain retailer for a petroleum product sells a random amount, X, each day. Suppose that X, measured in hundreds of gallons, has the probability density function

$$f_X(x) = \begin{cases} (3/8)x^2 & 0 \le x \le 2 \\ 0 & \text{elsewhere.} \end{cases}$$

The retailer's profit turns out to be 5 cents per gallon if $X \le 1$, and 8 cents per gallon for each extra gallon if $X > 1$. Find the retailer's expected profit for any given day.

3 Consider a random variable X with the following pdf:

$$f_X(x) = 1 - a|x|, \quad |x| \le 1/a$$

a. Find the constant a and compute the mean and the standard deviation of X.

b. The random variable X is applied to a "full-wave" rectifier whose output-input gain characteristic is $y = b|x|$. Determine the mean and standard deviation of the output random variable.

c. The random variable X is applied to a "half-wave" rectifier whose output-input gain characteristic is $y = bx, x \ge 0$, and $y = 0, x < 0$. Determine the mean and standard deviation of the output random variable.

4 For an arbitrary continuous random variable X with mean m and variance σ^2, find a reasonable lower bound on the probability of the event $\{|X - m| \le a\}$, for a an arbitrary positive constant. For what values of a is the bound nontrivial?

5 (a) A discrete random variable X takes values from the set $\mathcal{S}_X = \{-2, -1, 0, 1, 2\}$ with probabilities $P(X = i) = .125$, for $i = -2, -1, 1, 2$, and $P(X = 0) = .5$. A random variable Y is defined by

$$Y = g(X) = \begin{cases} \log_2 \frac{1}{P(X)} & \text{if } P(X) \neq 0\,, \\ 0 & \text{otherwise}\,, \end{cases}$$

where $P(X)$ is the probability of X. Find the mean of Y. (The mean of Y, which is also called the *entropy* of X, is a measure of information contained in X.)

(b) A repeater on a telephone line has a lightning protection circuit. When a lightening flash comes, the repeater has a probability p of surviving it. In a given period of time $(0, t)$, the number of lightning flashes follows a Poisson distribution with parameter λt. What is the probability that the repeater is still functioning after t seconds?