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Stochastic Processes, 94.553

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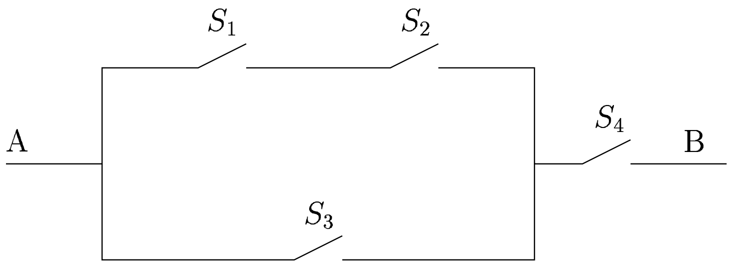
Problem Set #1

1 Given two events A and B, belonging to the same sample space, with $P(A) = 0.4$ and $P(B) = 0.7$. What are the maximum and minimum possible values for $P(A \cap B)$?

2 Consider the following connection of switches:



Define the event: $E_i, i = 1, 2, 3, 4$ as: Switch S_i is closed. Assume that $P(E_1) = P(E_2) = P(E_3) = P(E_4) = a$. Define the event $\mathcal{E}$ as the event that point A is connected to point B. Compute the probability of the event $\mathcal{E}$ if E_is are independent.

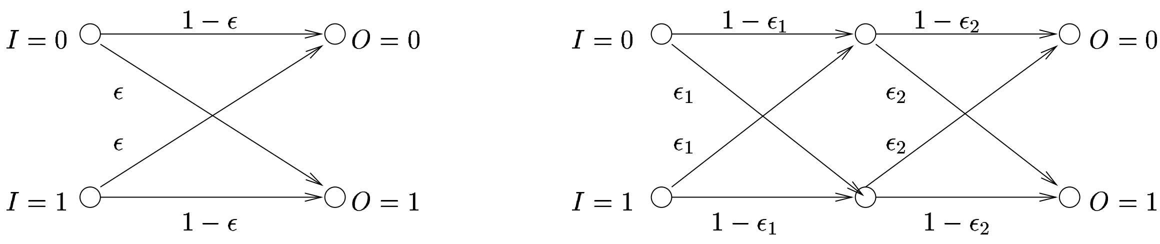
3 Six people including Mary and Mike receive gifts in a party. There are six gifts with values $\$10, \$20, \$30, \$40, \$50, \$60$ which are distributed at random among the recipients. If each of the 6 people receives one gift, (a) In how many (distinct) ways can the gifts be distributed? (b) Find the probability that Mary's gift is more expensive than \$25. (c) Find the probability that Mary's gift is at least \$20 more expensive than Mike's. (d) If there were three \$10 and three \$20 gifts, and the gifts were only distinguished by their price, in how many (distinct) ways could they be distributed among the 6 people such that every person has a gift? (e) Under the assumptions of part (d) and random distribution of gifts, find the probability that the values of Mary's and Mike's gifts are the same.

4 Two numbers are selected at random from the interval $[0, 1]$. Find the probability that they differ by more than $1/2$.

5 Russian roulette is played with a revolver equipped with a rotatable magazine of six shots. The revolver is loaded with one bullet. The first duelist, A, rotates the magazine at random, points the revolver at his head and presses the trigger. If, afterwards, he is still alive, he hands the revolver to the other duelist, B, who acts in the same way as A. The players shoot alternately in this manner, until a shot goes off. Determine the probability that A is killed.

6 Show that if A and B are independent events, then the pairs A and B^c, A^c and B, and A^c and B^c are also independent.

7 A binary symmetric communication channel (BSC) is modeled as in Figure (a), where $\epsilon = P(O = 1|I = 0) = P(O = 0|I = 1)$, and the notations "I" and "O" stand for the input and the output of the channel, respectively. To set up a (random) experiment, a group of students connect two BSC's with parameters ϵ_1 and ϵ_2 in cascade (as in Figure (b)). A bit is then sent over this composite channel with $P(I = 0) = p$, and the pair (I, O) of input and outp ut bits are observed. Assuming that the connection of the channels doesn't change the initial characteristics of any of them, and knowing th at the composite channel can also be modeled as a BSC, (a) Find the parameter ϵ for



(a) (b)

the composite channel (ϵ_t). (b) Error in transmission occurs if the output and the input of the channel are different. In the above experiment, find the probability of transmission error $P(e)$ in terms of ϵ_t. (c) If n bits are transmitted over the composite channel, find the probability that at most one bit is in error.

8 A binary asymmetric communication channel is modeled as in the following figure, where $\epsilon_1 = P(O = 1|I = 0)$, $\epsilon_2 = P(O = 0|I = 1)$, and the notations "I" and "O" stand for the input and the output of the channel, respectively. A bit is sent over this channel with $P(I = 0) = p$, and the pair (I, O) of input and output bits are observed. Error in transmission occurs if the output and the input of the channel are different. (a) Find the probability of transmission error $P(e)$ in terms of ϵ_1, ϵ_2 and p. (b) To have a more reliable communication over this channel, a repetition code of length 5 is used at the input. This means that each input bit is encoded to a block of five bits of the same type (0 or 1), before being sent over the channel. At the output, majority rule is used to decide on the corresponding output bit (the output bit is 1 if there are more ones in the block than there are zeros, and is 0 otherwise). Find $P(e)$ in this case. (c) For $p = \epsilon_1 = \epsilon_2 = 0.5$, compare the results of parts (a) and (b). Explain. (d) Using the repetition code described in part (b), a bit is sent over the channel. Knowing that the output block is 11011, find the probability that the transmitted bit is 1?